YouTube Lecture Links & Notes:
Unit Test -1 Last Minute Revision Notes for K Scheme: Topic - Integration
YT Link : https://www.youtube.com/watch?v=8ksVyzVeEKU

Integration

1) $\int 0 \cdot dx = \text{constant}$

2) $\int 1 \cdot dx = x + c$

3) $\int x^n dx = \frac{x^{n+1}}{n+1} + c$

4) $\int k \cdot x^n dx = k \cdot \int x^n dx$

5) $\int a^x dx = \frac{a^x}{\log_e a} + c$

6) $\int e^x \cdot dx = e^x + c$

7) $\int \frac{1}{x} dx = \log x + c$

8) $\int \log x \cdot dx = x \cdot \log x - x + c$

9) $\int x \cdot e^x dx = e^x (x-1) + c$

10) $\int \frac{1}{ax+b} dx = \log|ax+b| \cdot \frac{1}{a} + c$

11) $\int \sin x \cdot dx = -\cos x + c$

12) $\int \cos x \cdot dx = \sin x + c$

13) $\int \tan x \cdot dx = \log|\sec x| + c$

14) $\int \cot x \cdot dx = \log|\sin x| + c$

15) $\int \sec x \cdot dx = \log|\sec x + \tan x| + c$

16) $\int \operatorname{cosec} x \cdot dx = \log|\operatorname{cosec} x - \cot x| + c$

17) $\int \sec x \cdot \tan x \, dx = \sec x + c$

18) $\int \operatorname{cosec} x \cdot \cot x \, dx = -\operatorname{cosec} x + c$

19) $\int \sec^2 x \, dx = \tan x + c$

20) $\int \operatorname{cosec}^2 x \, dx = -\cot x + c$

21) $\int \frac{1}{\sqrt{1-x^2}} dx = \sin^{-1} x + c$

22) $\int \frac{-1}{\sqrt{1-x^2}} dx = \cos^{-1} x + c$

23) $\int \frac{1}{1+x^2} dx = \tan^{-1} x + c$

24) $\int \frac{-1}{1+x^2} dx = \cot^{-1} x + c$

25) $\int \frac{1}{x\sqrt{x^2-1}} dx = \sec^{-1} x + c$

26) $\int \frac{-1}{x\sqrt{x^2-1}} dx = \operatorname{cosec}^{-1} x + c$

27) Integration by Parts

$$\int u \cdot v \, dx = u \cdot \int v \, dx - \int \left[\frac{d}{dx} u \cdot \int v \cdot dx \right] dx$$

L = logarithmic
I = Inverse
A = Algebric = x
T = Trignometric
E = Exponential = e^x

$\int e^x \cdot x \, dx$

$\int \underset{u}{x} \cdot \underset{v}{e^x} \, dx$

Definite Integration

$$\text{Area} = \int_a^b y \cdot dx$$

$$\int_a^b f(x) \cdot dx = \int_a^b f(a+b-x) \cdot dx$$

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Q.1) find area enclosed between X axis & $y = 2x$ between Ordinate $x = 1$ & $x = 3$

$\Rightarrow$

$$\text{Area} = \int_a^b y \cdot dx$$

upper limit $= 3$

lower limit $= 1$

$$= \int_1^3 2x \cdot dx$$

$$= 2 \int_1^3 x \cdot dx$$

$$= 2 \left[\frac{x^{1+1}}{1+1} \right]_1^3 \quad \cdots \int x^n dx = \frac{x^{n+1}}{n+1} + C$$

$$= 2 \left[\frac{x^2}{2} \right]_1^3$$

$$= 2 \left[\frac{3^2}{2} - \frac{1^2}{2} \right]$$

$$= 2 \left[\frac{9}{2} - \frac{1}{2} \right]$$

$$= \cancel{2} \left[\frac{9-1}{\cancel{2}} \right]$$

$$\text{Area} = 8 \text{ sq. units}$$

Imp

Q2) $I = \int_0^{\pi/2} \frac{\sqrt{\sin x}}{\sqrt{\sin x} + \sqrt{\cos x}}\, dx$

$\rightarrow I = \int_0^{\pi/2} \frac{\sqrt{\sin x}}{\sqrt{\sin x} + \sqrt{\cos x}}\, dx$ ——— ①

$I = \int_0^{\pi/2} \frac{\sqrt{\sin(0+\pi/2-x)}}{\sqrt{\sin(0+\pi/2-x)} + \sqrt{\cos(0+\pi/2-x)}}\, dx$ $\quad \int_a^b f(x)\,dx = \int_a^b f(a+b-x)\,dx$

$= \int_0^{\pi/2} \frac{\sqrt{\sin(\pi/2-x)}}{\sqrt{\sin(\pi/2-x)} + \sqrt{\cos(\pi/2-x)}}\, dx$ $\quad \sin(\pi/2-x) = \cos x$, $\cos(\pi/2-x) = \sin x$

$I = \int_0^{\pi/2} \frac{\sqrt{\cos x}}{\sqrt{\cos x} + \sqrt{\sin x}}\, dx$ ——— ②

eq^n ① + eq^n ②

$I + I = \int_0^{\pi/2} \frac{\sqrt{\sin x}}{\sqrt{\sin x} + \sqrt{\cos x}}\, dx + \int_0^{\pi/2} \frac{\sqrt{\cos x}}{\sqrt{\cos x} + \sqrt{\sin x}}\, dx$

$2I = \int_0^{\pi/2} \left[\frac{\sqrt{\sin x}}{\underbrace{\sqrt{\sin x} + \sqrt{\cos x}}_{a\ +\ b}} + \frac{\sqrt{\cos x}}{\underbrace{\sqrt{\cos x} + \sqrt{\sin x}}_{b\ +\ a}} \right] dx$

$2I = \int_0^{\pi/2} \frac{\cancel{(\sqrt{\sin x} + \sqrt{\cos x})}^{\,1}}{\cancel{(\sqrt{\sin x} + \sqrt{\cos x})}_{\,1}}\, dx$

$= \int_0^{\pi/2} \frac{1}{1} \cdot dx$

$= \int_0^{\pi/2} 1 \cdot dx$

$2I = [x]_0^{\pi/2}$

$2I = \left[\frac{\pi}{2} - 0\right]$

$2I = \frac{\pi}{2}$

$I = \frac{\pi}{2} \times \frac{1}{2}$

$\boxed{I = \frac{\pi}{4}}$

Q 3)

$$I = \int \left[\frac{1}{1+x^2} + 5^x\right] dx$$

$$\Rightarrow \quad I = \int \frac{1}{1+x^2}\,dx + \int 5^x\,dx$$

$$= \tan^{-1}x + \frac{5^x}{\log 5} + c$$

$$\because \int \frac{1}{1+x^2}\,dx = \tan^{-1}x + c$$

$$\int a^x\,dx = \frac{a^x}{\log a} + c$$

Q.4) $I = \int \sin^2 x \cdot dx$

$= \int \frac{1}{2}(1 - \cos 2x)\,dx$

$= \frac{1}{2}\int (1 - \cos 2x)\,dx$

$= \frac{1}{2}\left[\int 1.dx - \int \cos 2x\,dx\right]$

$= \frac{1}{2}\left[x - (\sin 2x) \times \frac{1}{2}\right] + c$

$I = \frac{1}{2}\left[x - \frac{1}{2}\sin 2x\right] + c$

$\cos 2A = 1 - 2\sin^2 A$

$\cos 2x = 1 - 2\sin^2 x$

$2\sin^2 x = 1 - \cos 2x$

$\sin^2 x = \frac{1}{2}(1 - \cos 2x)$

$\int 1.dx = x + c$

$\int \cos x\,dx = \sin x + c$

Imp

Q. $I = \int \frac{e^x (x+1)\,dx}{\cos^2 (x \cdot e^x)}$

$$I = \int \frac{dt}{\cos^2 t}$$

$$= \int \frac{1}{\cos^2 t}\,dt$$

$$= \int \sec^2 t \cdot dt$$

$$= \tan t + C$$

$$I = \tan(x \cdot e^x) + C$$

$x \cdot e^x = t$

diff. w.r.t 'x'

$$\frac{d}{dx}(\underset{u}{x} \cdot \underset{v}{e^x}) = \frac{d}{dx} t$$

$$\frac{d}{dx} u \cdot v = u \cdot \frac{d}{dx} v + v \cdot \frac{d}{dx} u$$

$$x \cdot \frac{d}{dx} e^x + e^x \cdot \frac{d}{dx} x = \frac{dt}{dx}$$

$$x e^x + e^x \cdot 1 = \frac{dt}{dx}$$

$$e^x (x+1) = \frac{dt}{dx}$$

$$e^x (x+1) \cdot dx = dt$$

Imp

Q5) $I = \int \frac{1}{2+3\cos x} \cdot dx$

$\tan\left(\frac{x}{2}\right) = t$

$\cos x = \frac{1-t^2}{1+t^2}$

$dx = \frac{2 \cdot dt}{1+t^2}$

$\Rightarrow$

$I = \int \frac{1}{\left[2+3\left(\frac{1-t^2}{1+t^2}\right)\right]} \cdot \left(\frac{2\,dt}{1+t^2}\right)$

$= \int \frac{1/1}{\frac{2(1+t^2)+3(1-t^2)}{1(1+t^2)}} \left(\frac{2\,dt}{1+t^2}\right)$

$= \int \frac{1\cancel{(1+t^2)}}{[2(1+t^2)+3(1-t^2)]} \cdot \frac{2\,dt}{\cancel{(1+t^2)}}$

$= \int \frac{2\,dt}{2(1+t^2)+3(1-t^2)}$

$= \int \frac{2\,dt}{2\times1+2\times t^2+3\times1-3\times t^2}$

$= \int \frac{2\,dt}{2+2t^2+3-3t^2}$

$= 2\int \frac{1}{5-t^2} \cdot dt$

$= \cancel{2}\left[\frac{1}{\cancel{2}\times\sqrt{5}} \cdot \log\left|\frac{\sqrt{5}+t}{\sqrt{5}-t}\right|\right] + c$

$\int \frac{1}{a^2-x^2}\,dx = \frac{1}{2a} \cdot \log\left|\frac{a+x}{a-x}\right| + c$

$x^2 = t^2 \Rightarrow x = t$

$a^2 = 5 \Rightarrow a = \sqrt{5}$

$= \frac{1}{\sqrt{5}} \cdot \log\left|\frac{\sqrt{5}+\tan(x/2)}{\sqrt{5}-\tan(x/2)}\right| + c$

Imp Q. $I = \int x \tan^{-1}x \, dx$

$\Rightarrow \int u \cdot v \, dx = u \int v \cdot dx - \int \left[\frac{d}{dx} u \cdot \int v \cdot dx \right] dx$

L
I = $\tan^{-1}x = u$
A = $x = v$
T
E

$\int \tan^{-1}x \cdot x \, dx = \tan^{-1}x \int x^1 dx - \int \left[\frac{d}{dx} \tan^{-1}x \int x \cdot dx \right] dx$

$= \tan^{-1}x \cdot \left[\frac{x^{1+1}}{1+1} \right] - \int \left[\frac{1}{1+x^2} \cdot \left(\frac{x^{1+1}}{1+1} \right) \right] dx + C$

$\int x^n dx = \frac{x^{n+1}}{n+1} + C$

$\frac{d}{dx} \tan^{-1}x = \frac{1}{1+x^2}$

$= \tan^{-1}x \cdot \frac{x^2}{2} - \int \left[\frac{1}{1+x^2} \cdot \frac{x^2}{2} \right] dx + C$

$I = \tan^{-1}x \cdot \frac{x^2}{2} - \frac{1}{2} \underbrace{\boxed{\int \left[\frac{x^2}{1+x^2} \right] dx}}_{I_1} + C$ ——— (1)

$I_1 = \int \frac{x^2}{1+x^2} dx$

$= \int \frac{x^2}{x^2+1} dx$

$= \int \frac{(x^2+1)-1}{x^2+1} \cdot dx$

$= \int \left[\frac{\cancel{(x^2+1)}}{\cancel{(x^2+1)}} - \frac{1}{x^2+1} \right] dx$

$= \int \left[1 - \frac{1}{x^2+1} \right] dx$

$= \int 1 \cdot dx - \int \frac{1}{1+x^2} \cdot dx$

$I_1 = x - \tan^{-1}x + C$ //

eqn (1) $\Rightarrow$ $I = \tan^{-1}x \cdot \frac{x^2}{2} - \frac{1}{2} \left[x - \tan^{-1}x \right] + C$ //

$I = \frac{x^2}{2} \cdot \tan^{-1}x - \frac{1}{2} \left(x - \tan^{-1}x \right) + C$ //

Q. $I = \int (x^m + m^x + m^m + e^x)\,dx$

$= \int x^m\,dx + \int m^x\,dx + \int m^m\,dx + \int e^x\,dx$

$= \frac{x^{m+1}}{m+1} + \frac{m^x}{\log_e m} + m^m \int 1.dx + e^x + c$

$I = \frac{x^{m+1}}{m+1} + \frac{m^x}{\log_e m} + m^m . x + e^x + c$ //

$\int x^n . dx = \frac{x^{n+1}}{n+1} + c$

$\int a^x dx = \frac{a^x}{\log_e a} + c$

$\int 1 . dx = x + c$

$\int e^x . dx = e^x + c$

Q. $I = \int (e^x + x^e + e^e + \log x) . dx$

$\Rightarrow I = \int e^x . dx + \int x^e . dx + \int e^e dx + \int \log x . dx$

$= e^x + \frac{x^{e+1}}{e+1} + e^e \int 1 . dx + x . \log x - x + C$

$I = e^x + \frac{x^{e+1}}{e+1} + e^e . x + x . \log x - x + c$ //

$\int e^x dx = e^x + c$

$\int x^n dx = \frac{x^{n+1}}{n+1} + c$

$\int 1 . dx = x + c$

$\int \log x\, dx = x . \log x - x + c$

Imp

Q. $I = \int_1^2 \frac{1}{3x-2}\,dx$

$= \left[\log|3x-2| . \frac{1}{3} \right]_1^2$

$\int \frac{1}{ax+b} dx = \log|ax+b| . \frac{1}{a} + c$

$= \left[\frac{1}{3} . \log|3x-2| \right]_1^2$

$= \left[\frac{1}{3} . \log|3\times2-2| - \frac{1}{3} . \log|3\times1-2| \right]$

$= \frac{1}{3} . \log|4| - \frac{1}{3} . \log|1|$

$= \frac{1}{3} . \log|4| - \frac{1}{3} \times 0$

$I = \frac{1}{3} . \log|4|$ //

Imp

Q. $I = \int \frac{1 \cdot \log x}{x(2+\log x)(3+\log x)} \, dx$

$\log x = t$

diff. w.r.t 't'

$\frac{d}{dx} \log x = \frac{d}{dx} t$

$\frac{1}{x} = \frac{dt}{dx}$

$\frac{1}{x} \cdot dx = dt$

$I = \int \frac{t}{(2+t) \cdot (3+t)} \, dt$

$\frac{t}{(2+t)(3+t)} = \frac{A}{(2+t)} + \frac{B}{(3+t)}$

$t = A(3+t) + B(2+t)$ ——— ①

$3+t=0, \; t=-3 \Rightarrow$

$-3 = A(3-3) + B(2-3)$

$-3 = A(0) + B(-1)$

$-3 = 0 + B(-1)$

$-3 = B(-1)$

$\frac{-3}{-1} = B$

$\boxed{B = 3}$

$2+t=0, \; t=-2$

$-2 = A(3-2) + B(2-2)$

$-2 = A(1) + B(0)$

$-2 = A + 0$

$\boxed{-2 = A}$

$\frac{t}{(2+t)(3+t)} = \frac{-2}{(2+t)} + \frac{3}{(3+t)}$

$I = \int \left[\frac{-2}{2+t} + \frac{3}{3+t} \right] \cdot dt$

$= \int \frac{-2}{2+t} \, dt + \int \frac{3}{3+t} \, dt$

$= -2 \int \frac{1}{1t+2} \, dt + 3 \int \frac{1}{1t+3} \, dt$

$= -2\left(\log|t+2| \cdot \frac{1}{1} \right) + 3\left(\log|t+3| \cdot \frac{1}{1} \right) + C$

$\int \frac{1}{ax+b} \, dx = \log|ax+b| \cdot \frac{1}{a} + C$

$I = -2 \cdot \log|t+2| + 3 \cdot \log|t+3| + C$ //

$I = -2 \cdot \log|\log x + 2| + 3 \cdot \log|\log x + 3| + C$ //

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